

Math 323 - Formal Mathematical Reasoning and Writing
Problem Session
Wednesday, 2/4/15

1. Each of the following items asks you to give either a definition or a statement of a theorem. Please give **mathematically precise** statements.
 - (a) The integers \mathbb{Z} has *trichotomy*. State precisely what this means.
 - (b) State what it means for a number $C \in \mathbb{Z}$ to be a *multiplicative identity* for \mathbb{Z} .
 - (c) Give a precise statement of the Division Algorithm for \mathbb{Z} .
2. Prove that $(n + 1)! \geq 2^n$ for any integer $n \geq 1$.
3. Prove that for every integer x , if x is odd then there exists an integer y such that $x^2 = 4y + 1$. (For a slightly more challenging problem, change the '4' to an '8'.)
4. ¹Decide whether the following statements are true or false. If the statement is true, prove it. If the statement is false, give a counterexample to show that it is false.
 - (a) For all integers x, y , if $xy > 0$ then $x^2 + y^2 > 0$.
 - (b) For all integers x, y , if $x^2 + y^2 > 0$ then $xy > 0$.

If you've finished, here's a bonus problem!

- ✈ Prove that for every integer $n \geq 0$, there exists an integer A such that $9^n - 1 = 8A$.

¹One of these is easy. The other one is hard.